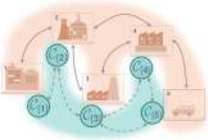
Optimal control for large-scale dynamical systems

Target challenges

1) Optimality in distributed tasks



- *Myopic,* local improvement→ global effect

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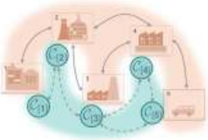

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- *Noisy* data → optimal control policy
- Tension: performance VS guarantees

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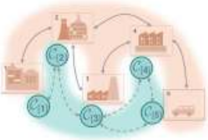

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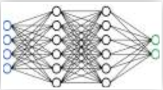
3) Guarantees of learning-based control



- Neural network policies, nonlinear objectives
- <u>Closed-loop stability? Safety?</u>

Selected Contributions: Part 1

Optimal Distributed Control (ODC) for Linear Systems

[1] “*Sparsity invariance for convex design of distributed controllers*”, Furieri, Zheng, Papachristodoulou, Kamgarpour, TCNS, 2020

[2] “*Learning the globally optimal distributed LQ regulator*”, Furieri, Zheng, Kamgarpour, L4DC, 2020

Beyond long-standing limitations of linear ODC[1]

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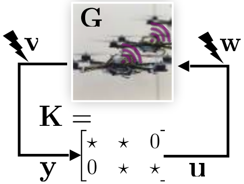

$$= \|(\mathbf{w},\mathbf{v}) \underset{\mathbf{G},\mathbf{K}}{\rightarrow} (\mathbf{u},\mathbf{y})\|_{\mathcal{H}_2/\mathcal{H}_\infty}$$

$$\min_{\mathbf{K}\ \text{stabilizing}} \quad \text{cost}(\mathbf{G},\mathbf{K})$$

$$\text{subject to} \quad \mathbf{K} = \begin{bmatrix} \star & \star & 0 \\ 0 & \star & \star \end{bmatrix}$$

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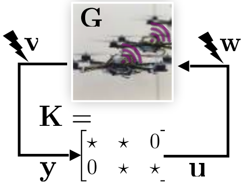

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Limitation: convex reformulation available only if *Quadratic Invariance* (QI) holds [Rotkowitz et al., 2006]

"*design sparse closed-loop maps*"

"*sparse controller*" $\Longleftrightarrow$ "*sparse closed-loop maps*"

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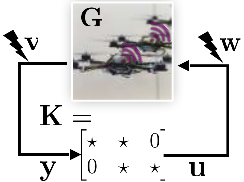

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My Contribution: near-optimal convex restrictions

1. QI as a special case of **Sparsity Invariance (SI)** ← Plant-independent
2. **Globally optimal and sparse** $\mathbf{K}$ for some non-QI cases
3. Near-optimality guarantees **for arbitrary sparsity patterns**

$\Longrightarrow$ **QI no-longer a limitation for sparse controller design**

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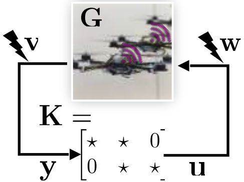

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Next question: unknown system model?

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Learning-based control philosophies

Model-based

Model-free
(RL-like)

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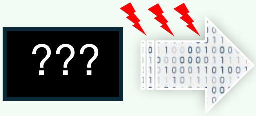

Noisy trajectories

System Identification

Optimal Control

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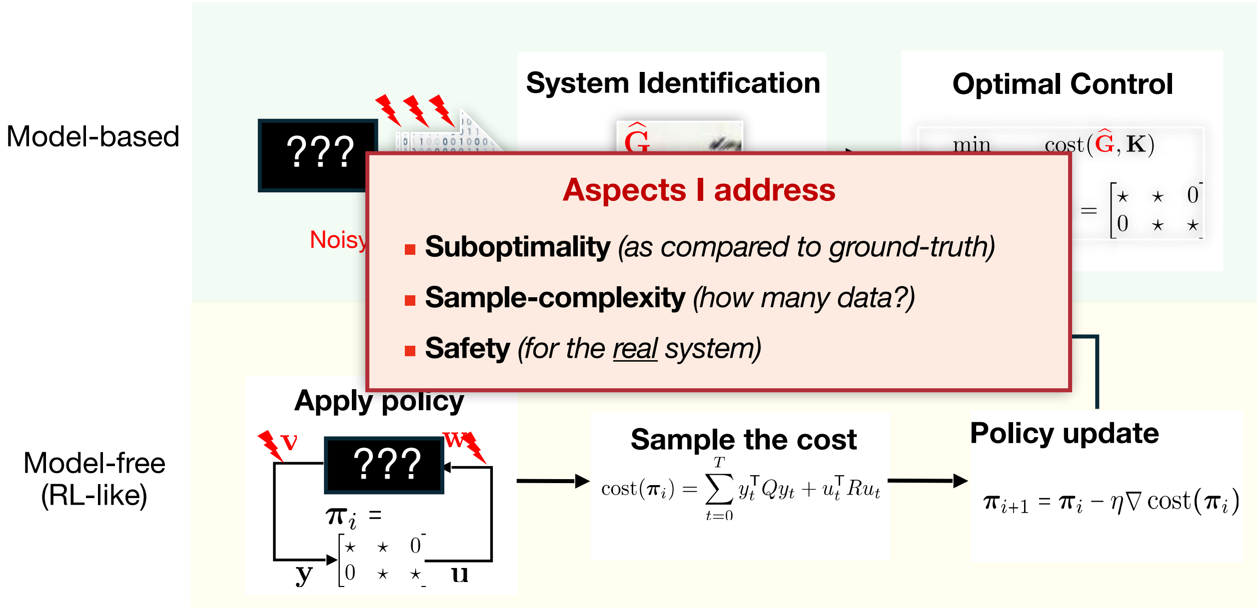

Aspects I address

- **Suboptimality** *(as compared to ground-truth)*
- **Sample-complexity** *(how many data?)*
- **Safety** *(for the <u>real</u> system)*

Model-free learning of the globally optimal distributed LQ regulator[1]

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Result: Favorable optimization landscape[1]

The cost is nonconvex in distributed policies $\pi \in \begin{bmatrix} \star & \star & 0 \\ 0 & \star & \star \end{bmatrix}$. However;

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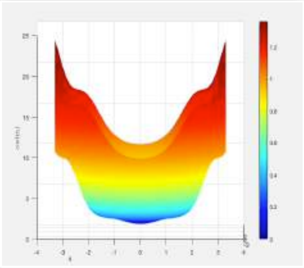
- Several non-QI cases are also PL!
- *… PL holds for arbitrary sparsities?* No.



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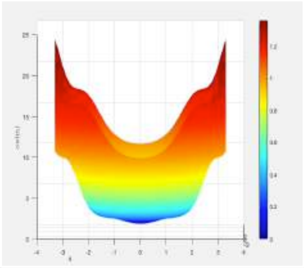

Result: Sample-complexity for global optimality[1]

Assume that PL holds. Run model-free policy gradient for T steps with

$$T \propto \left(\mathrm{card}\left(\begin{bmatrix} \star & \star & 0 \\ 0 & \star & \star \end{bmatrix} \right)^2 \epsilon^{-2} \, \delta^{-4} \right)$$

Then: $\mathrm{cost}(\boldsymbol{\pi}_T) - \mathrm{cost}(\boldsymbol{\pi}^\star) \leq \epsilon$

with high probability $1 - \delta$.

Selected Contributions: Part 2

Learning to control with stability guarantees for nonlinear systems

[1] Furieri, Galimberti, F. Trecate, "*Neural system level synthesis: learning over all stabilizing policies for nonlinear systems*", [CDC 2022]
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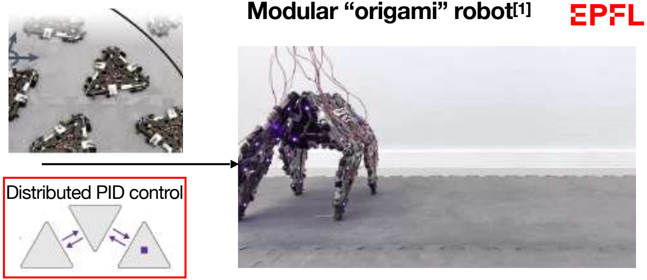
- Complex real-world systems are nonlinear
- Frequent availability of stabilizing controllers around equilibrium or a reference

Modular "origami" robot[1]



All-terrain legged-robots[2]

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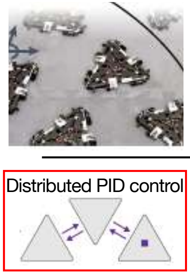

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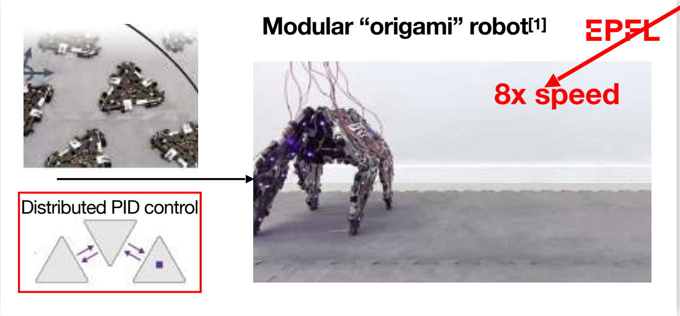

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Distributed PID control

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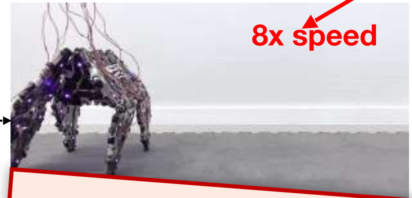

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Improve performance without compromising stability?

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Parametrization of all nonlinear stabilizing controllers[1]

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$$\begin{cases} x_t = f_t(x_{t-1}, u_{t-1}) + w_t \\ u_t = K_t(x_{t:0}) \end{cases}$$

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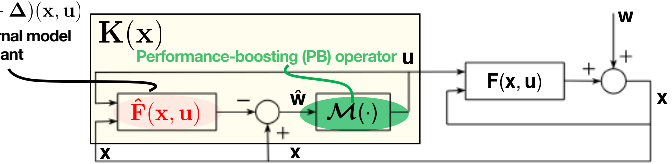
$\widehat{\mathbf{F}}(\mathbf{x}, \mathbf{u}) = (\mathbf{F} + \mathbf{\Delta})(\mathbf{x}, \mathbf{u})$

Uncertain Internal model of the plant



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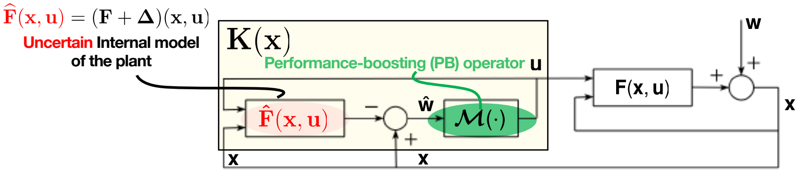

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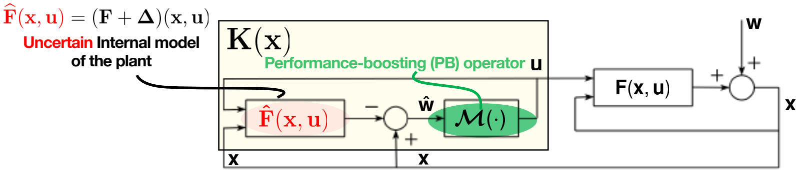

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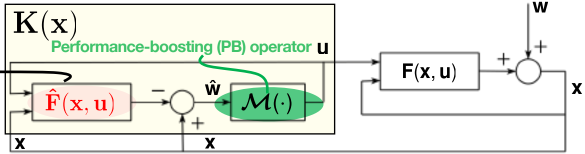

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Importance of main result for nonlinear optimal control

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Nonlinear generalization of "Youla", "System Level Synthesis",…

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- **Theory:** directly optimize over stable closed-loop operator $\mathcal{M}$
- **Implementation:** compatible with **deep neural network** stable $\mathcal{M}$ operators![3]

TensorFlow
PyTorch

[3] *"Recurrent equilibrium networks: Flexible dynamic models with guaranteed stability and robustness"*, Revay, Wang, Manchester, [TAC23]

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[2] Furieri, Galimberti, F. Trecate, *"Learning to boost the performance of stable nonlinear systems"*, [ArXiV, 2024]

Nonlinear generalization of "Youla", "System Level Synthesis",…

$$\max_{\mathbf{K} \text{ is stabilizing}} \mathbb{E}_{\mathbf{w}}[\text{performance}(\mathbf{x},\mathbf{u})]$$
$$\text{subject to} \quad x_t = f_t(x_{t-1}, u_{t-1}) + w_t,$$
$$u_t = K_t(x_{t:0}), \quad t = 1, 2, \ldots$$

$\Longleftrightarrow$

$$\max_{\mathcal{M} \text{ is stable}} \mathbb{E}_{\mathbf{w}}[\text{performance}(\mathbf{x},\mathbf{u})]$$
$$\text{subject to} \quad x_t = f_t(x_{t-1}, u_{t-1}) + w_t,$$
$$\hat{w}_t = x_t - f_t(x_{t-1}, u_{t-1})$$
$$u_t = \mathcal{M}_t(\hat{w}_{t:0}), \quad t = 1, 2, \ldots$$

- **Theory:** directly optimize over stable closed-loop operator $\mathcal{M}$
- **Implementation:** compatible with **deep neural network** stable $\mathcal{M}$ operators![3]

TensorFlow
PyTorch

[3] *"Recurrent equilibrium networks: Flexible dynamic models with guaranteed stability and robustness"*, Revay, Wang, Manchester, [TAC23]

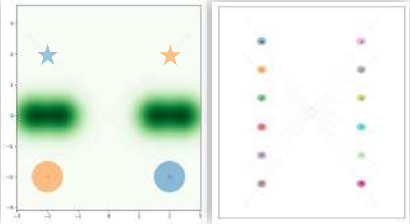
Remark: results compatible with *distributed* nonlinear control

[4] Massai, Saccani, Furieri, F. Trecate, «*Unconstrained learning of **networked nonlinear systems** via free parametrization of stable interconnected operators*», [ECC24]

[5] Saccani, Massai, Furieri, F. Trecate, «***Optimal distributed control** with stability guarantees by training a network of neural closed-loop maps*», *[ArXiV 2024]*

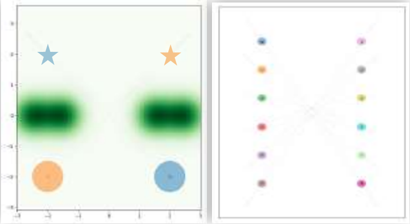
Numerical experiments: mobile robots

…before performance-boosting



Numerical experiments: mobile robots

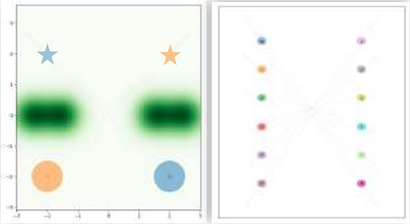
…before performance-boosting



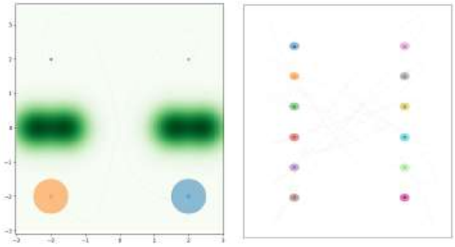
Numerical experiments: mobile robots

$$\text{cost}(\mathbf{x},\mathbf{u}) = \text{cost}_{\text{target}}(\mathbf{x},\mathbf{u}) + \text{cost}_{\text{collisions}}(\mathbf{x})$$

…before performance-boosting



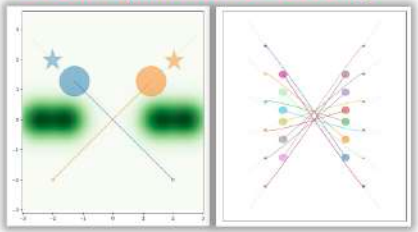
After performance-boosting

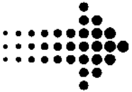


Numerical experiments: mobile robots

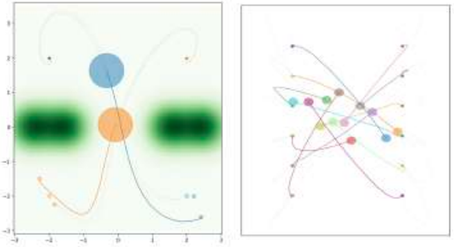
$$\text{cost}(\mathbf{x}, \mathbf{u}) = \text{cost}_{\text{target}}(\mathbf{x}, \mathbf{u}) + \text{cost}_{\text{collisions}}(\mathbf{x})$$

...before performance-boosting



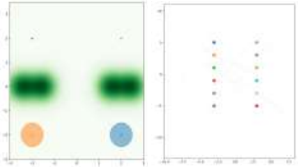


After performance-boosting



CL stability guaranteed even with partial training

25% training



75% training