# Closing the loop between nonlinear distributed control and learning-based optimization


## Dr. Luca Furieri


Swiss National Science Foundation

EPFL

ETH zürich

- SNSF Ambizione Fellow at EPF Lausanne (Jan. 2023 to present)
  - Principal investigator of «*Reliable Machine Learning for Distributed Control*»
- Postdoc at EPF Lausanne (Nov. 2020 to Dec. 2022)
  - Working with Prof. Giancarlo Ferrari Trecate
- PhD at ETH Zurich (Nov. 2016 to Sep 2020)
  - Supervised by Prof. Maryam Kamgarpour

# Optimal control for large-scale dynamical systems

## Target challenges

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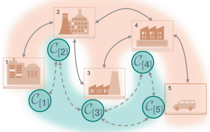
### 1) Optimality in distributed tasks




- *Myopic,* local improvement→ global effect

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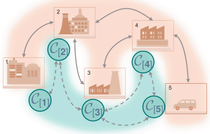


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- Tension: performance VS guarantees

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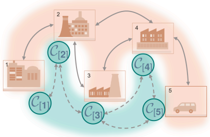

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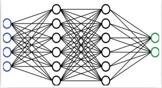
### 3) Guarantees of learning-based control



- Neural network policies, nonlinear objectives
- Closed-loop stability? Safety?

# Selected Contributions: Part 1

**Optimal Distributed Control (ODC) for Linear Systems**

[1] "*Sparsity invariance for convex design of distributed controllers*", Furieri, Zheng, Papachristodoulou, Kamgarpour, TCNS, 2020

[2] "*Learning the globally optimal distributed LQ regulator*", Furieri, Zheng, Kamgarpour, L4DC, 2020

# Beyond long-standing limitations of linear ODC[1]

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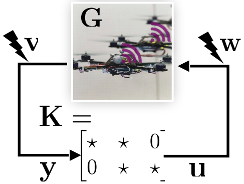


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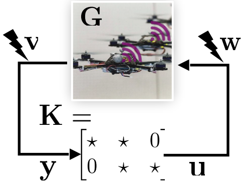


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"*design sparse closed-loop maps*"

"*sparse controller*" $\Longleftrightarrow$ "*sparse closed-loop maps*"

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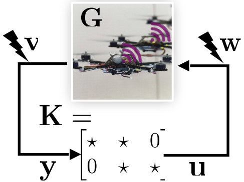


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### My Contribution: near-optimal convex restrictions

1. QI as a special case of **Sparsity Invariance (SI)** ← Plant-independent
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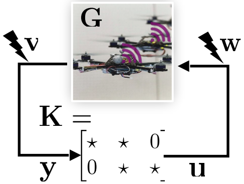


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**Next question: unknown system model?**

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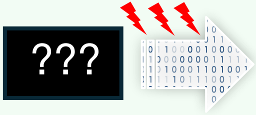


Noisy trajectories

**System Identification**


**Optimal Control**

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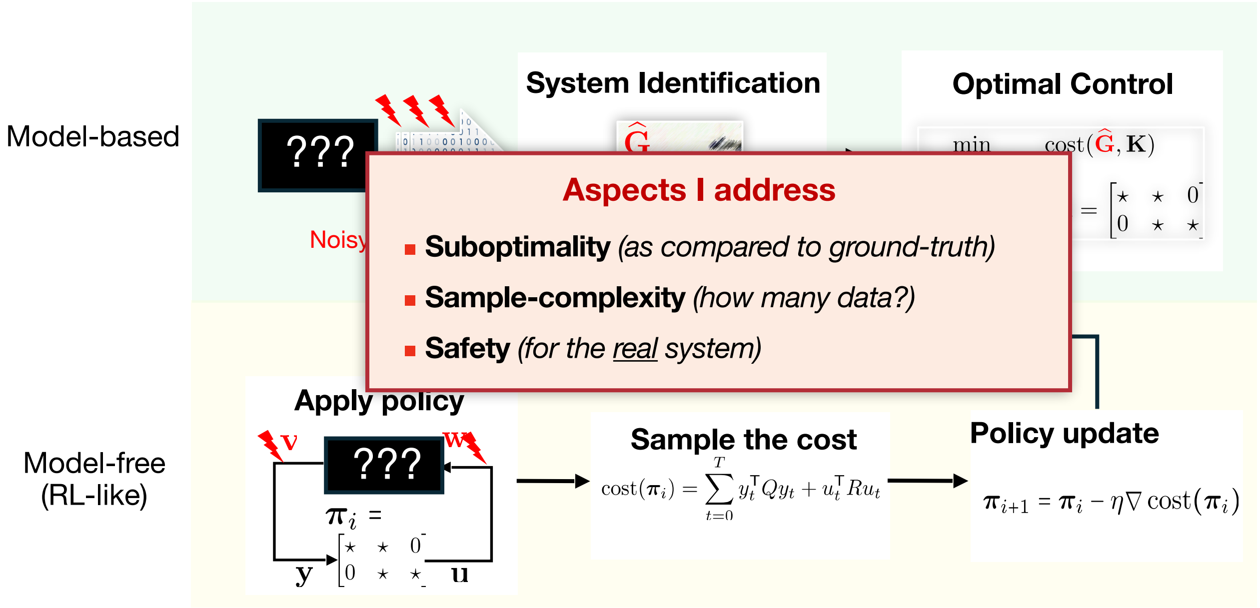


## Aspects I address

- **Suboptimality** *(as compared to ground-truth)*
- **Sample-complexity** *(how many data?)*
- **Safety** *(for the real system)*

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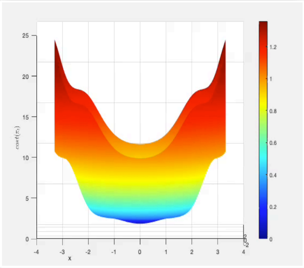
- … *PL holds for arbitrary sparsities?* No.



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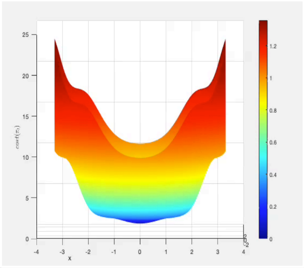

## Result: Sample-complexity for global optimality[1]

Assume that PL holds. Run model-free policy gradient for $T$ steps with

$$T \propto \left( \mathrm{card}\left( \begin{bmatrix} \star & \star & 0 \\ 0 & \star & \star \end{bmatrix} \right)^2 \epsilon^{-2}\, \delta^{-4} \right)$$

Then: $\mathrm{cost}(\boldsymbol{\pi}_T) - \mathrm{cost}(\boldsymbol{\pi}^\star) \leq \epsilon$

with high probability $1 - \delta$.

# Selected Contributions: Part 2

## Learning to control with stability guarantees for nonlinear systems

[1] Furieri, Galimberti, F. Trecate, “*Neural system level synthesis: learning over all stabilizing policies for nonlinear systems*”, [CDC 2022]
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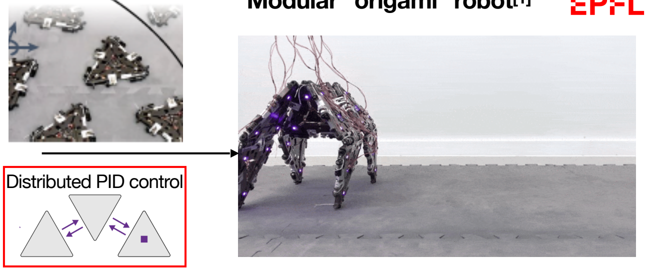
- Complex real-world systems are nonlinear
- Frequent availability of stabilizing controllers around equilibrium or a reference

**Modular "origami" robot[1]**




**All-terrain legged-robots[2]**



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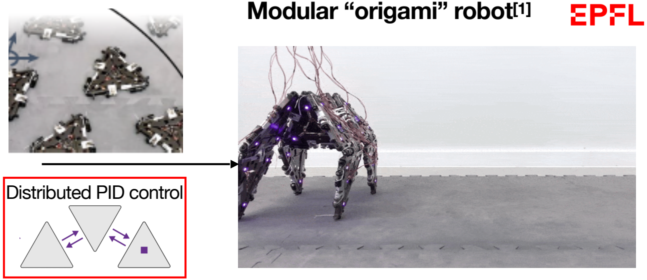


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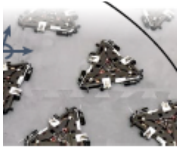

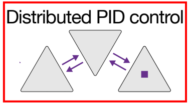


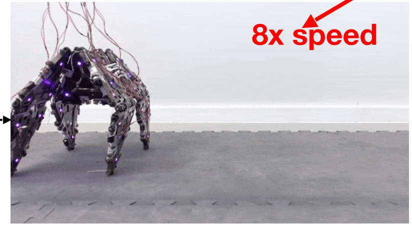

**8x speed**

**All-terrain legged-robots[2]** UNIVERSITY OF OXFORD


**>4x speed**

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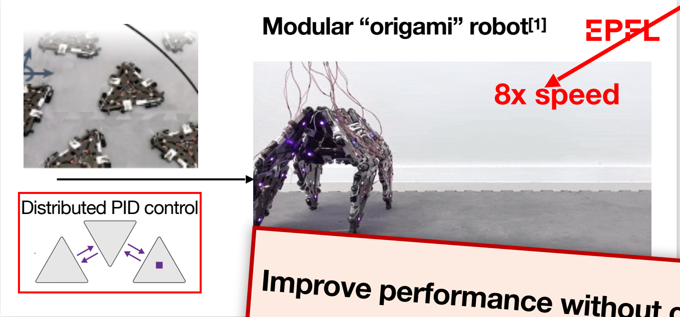



**Improve performance without compromising stability?**

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$$\begin{cases} x_t = f_t(x_{t-1}, u_{t-1}) + w_t \\ u_t = K_t(x_{t:0}) \end{cases}$$

$$\mathbf{x} = (x_0, x_1, \ldots)$$
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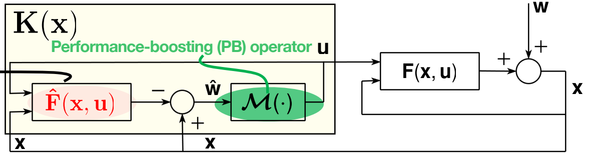
## Main result:[1,2] Parametrization of robustly stabilizing controllers + completeness

$\widehat{\mathbf{F}}(\mathbf{x}, \mathbf{u}) = (\mathbf{F} + \mathbf{\Delta})(\mathbf{x}, \mathbf{u})$

**Uncertain Internal model of the plant**

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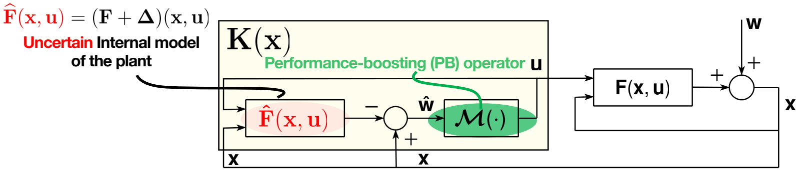


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- **Sufficiency:** the controller $\mathbf{K}(\mathbf{x})$ stabilizes the real system $\mathbf{F}(\mathbf{x},\mathbf{u})$ $\quad \text{gain}(\mathcal{M}) < \dfrac{1}{\text{gain}(\boldsymbol{\Delta})(\text{gain}(\mathcal{F})+1)}$

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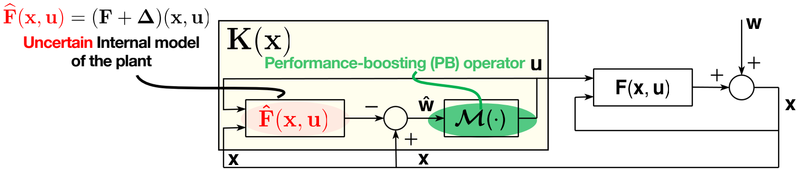


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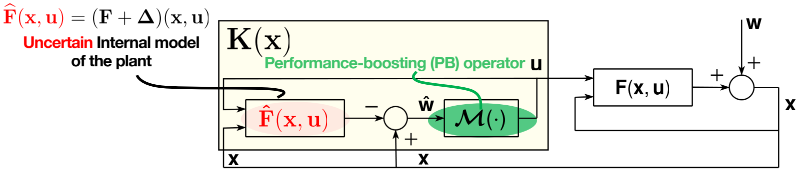


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# Importance of main result for nonlinear optimal control

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Nonlinear generalization of "Youla", "System Level Synthesis",…

$$
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\begin{aligned}
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\text{subject to} \quad & x_t = f_t(x_{t-1}, u_{t-1}) + w_t, \\
& \hat{w}_t = x_t - f_t(x_{t-1}, u_{t-1}) \\
& u_t = \mathcal{M}_t(\hat{w}_{t:0}), \quad t = 1, 2, \ldots
\end{aligned}
$$

- **Theory:** directly optimize over stable closed-loop operator $\mathcal{M}$

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[2] Furieri, Galimberti, F. Trecate, *"Learning to boost the performance of stable nonlinear systems"*, [ArXiV, 2024]

Nonlinear generalization of "Youla", "System Level Synthesis",…

$$\boxed{\begin{aligned} \max_{\mathbf{K} \text{ is stabilizing}} \quad & \mathbb{E}_{\mathbf{w}}\left[\text{performance}(\mathbf{x}, \mathbf{u})\right] \\ \text{subject to} \quad & x_t = f_t(x_{t-1}, u_{t-1}) + w_t, \\ & u_t = K_t(x_{t:0}), \quad t = 1, 2, \ldots \end{aligned}} \iff \boxed{\begin{aligned} \max_{\mathcal{M} \text{ is stable}} \quad & \mathbb{E}_{\mathbf{w}}\left[\text{performance}(\mathbf{x}, \mathbf{u})\right] \\ \text{subject to} \quad & x_t = f_t(x_{t-1}, u_{t-1}) + w_t, \\ & \hat{w}_t = x_t - f_t(x_{t-1}, u_{t-1}) \\ & u_t = \mathcal{M}_t(\hat{w}_{t:0}), \quad t = 1, 2, \ldots \end{aligned}}$$

- **Theory:** directly optimize over stable closed-loop operator $\mathcal{M}$
- **Implementation:** compatible with **deep neural network** stable $\mathcal{M}$ operators![3]

TensorFlow
PyTorch

[3] *"Recurrent equilibrium networks: Flexible dynamic models with guaranteed stability and robustness"*, Revay, Wang, Manchester, [TAC23]

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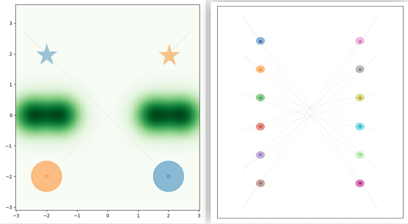
## Remark: results compatible with *distributed* nonlinear control

[4] Massai, Saccani, Furieri, F. Trecate, «*Unconstrained learning of* ***networked nonlinear systems*** *via free parametrization of stable interconnected operators*», [ECC24]

[5] Saccani, Massai, Furieri, F. Trecate, «***Optimal distributed control*** *with stability guarantees by training a network of neural closed-loop maps», [ArXiV 2024]*

# Numerical experiments: mobile robots

**…before performance-boosting**



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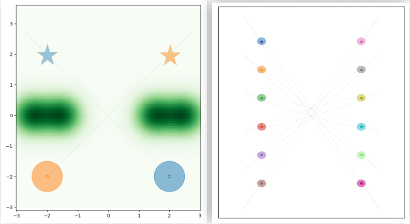

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$$\mathrm{cost}(\mathbf{x},\mathbf{u}) = \mathrm{cost}_{\mathrm{target}}(\mathbf{x},\mathbf{u}) + \mathrm{cost}_{\mathrm{collisions}}(\mathbf{x})$$

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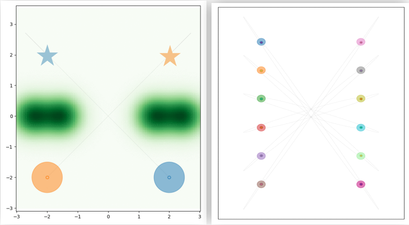
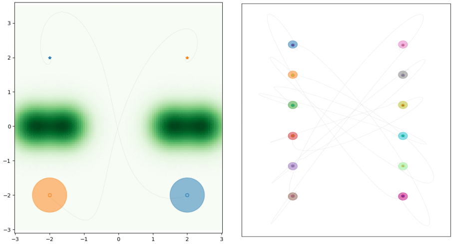


**After performance-boosting**



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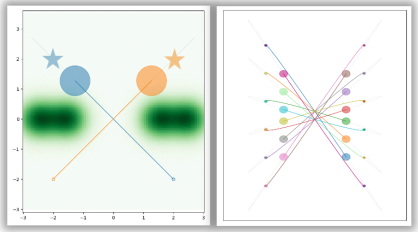

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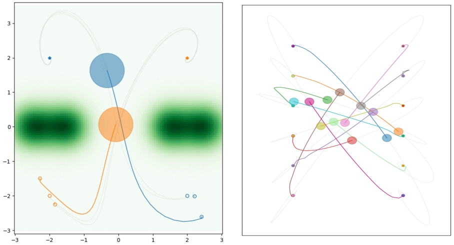
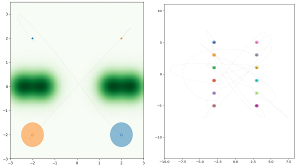


**CL stability guaranteed even with partial training**

25% training



75% training